Math 2411 **Quiz Two** **September 26, 2000**

You may use any books, notes, tables, calculators, or computers you wish. Write your answers so that someone other than yourself can understand your exposition. Please do not return this sheet.

1. Let f be the function defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

a)Find the partial derivatives of f at $(0,0)$ or explain carefully why they do not exist.

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \left(\frac{f(0+h) - f(0,0)}{h} \right) = \lim_{h \to 0} \left(\frac{0-0}{h} \right) = 0.$$

From the symmetry of f, we see that $\frac{\partial f}{\partial y}(0,0) = 0$, also.

b)Are the partial derivatives continuous at $(0,0)$? Explain.

For $(x,y) \neq (0,0)$, we have

$$\frac{\partial f}{\partial x}(x,y) = \frac{(x^2+y^2)y - xy2x}{(x^2+y^2)^2} = \frac{y^3 - x^2y}{(x^2+y^2)^2}.$$

To check for continuity at $(0,0)$, we need to see if

$$\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y) = \frac{\partial f}{\partial x}(0,0) = 0$$

Note that

$$\frac{\partial f}{\partial x}(0,y) = \frac{y^3}{y^4} = \frac{1}{y},$$

so that $\frac{\partial f}{\partial x}(x,y)$ can be made as large as you wish by taking $(0,y)$ close enough to $(0,0)$. Thus $\lim_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}(x,y)$ does not exist, and so $\frac{\partial f}{\partial x}(x,y)$ is **not continuous** at $(0,0)$. A very similar argument will show that $\frac{\partial f}{\partial y}(x,y)$ is also not continuous at $(0,0)$.

2. In the picture below, the curve is a level curve of the function $f(x,y)$, and one of the vectors shown is the gradient of *f*. The directional derivative of *f* in the direction of the vector **C** is negative.

a)Which of the vectors is the gradient of *f* ? Explain.

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The gradient must be normal to the level curve, so it must be either **B** or **G**. The directional derivative in the direction of **C** is the scalar product of the gradient with a vector having the direction of **C**. This is negative only if the cosine of the angle between the two is negative, which happens only if the angle is greater than $\pi/2$. Thus **G is the gradient**.

b)Which is larger, the directional derivative of *f* in the direction of **A** or the directional derivative in the direction of **F** ? Explain.

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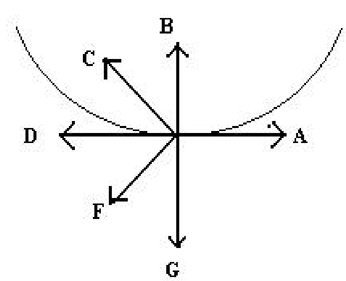
Again, the directional derivative in a direction **u** is the scalar product of the gradient with a vector in the direction **u**, and this will be bigger where the cosine of the angle between the two is bigger, or where the angle is smaller. Clearly the angle between the gradient **G** and the vector **F** is smaller than the angle between **G** and **A**. Thus the directional derivative in the direction of **F** is larger than the directional derivative in the direction of **A**.



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