Math 2411 **Quiz Four** **October 26, 2000**

You may use any books, notes, tables, calculators, or computers you wish. Write your answers so that someone other than yourself can understand your exposition. Please do not return this sheet.

1. The centroid of the plane region Ω is $\overline{x} = (4,10)$, and $\iint_{\Omega}(2x+5y)dA = 41$.

Find the area of Ω or explain carefully why it is impossible to do so from the information given.

We know that

$$\frac{\iint_{\Omega} xdA}{A} = 4 \text{ and } \frac{\iint_{\Omega} ydA}{A} = 10,$$

where A is the area of Ω.
Thus,

$$\iint_{\Omega} xdA = 4A \text{ and } \iint_{\Omega} ydA = 10A,$$

and so

$$\iint_{\Omega}(2x+5y)dA = 2\iint_{\Omega} xdA + 5\iint_{\Omega} ydA = 2(4A) + 5(10A)$$
$$= 58A = 41$$

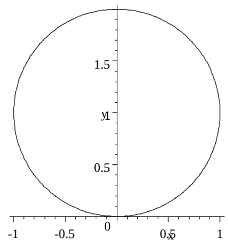
Hence,

$$A = \frac{41}{58}.$$

2. Let **S** be the solid bounded above by the surface $z = x^2 + y^2$ and below by the plane $z = 2y$. Supply the limits of integration:

a) $\iiint_{\mathbf{S}} f(x,y,z)dV = \int_{[?]}^{[?]}\int_{[?]}^{[?]}\int_{[?]}^{[?]} f(x,y,z)dzdydx.$

Here the first, or "inside" integration is with respect to z, so we have projected **S** onto the x-y plane. To see what the projection looks like, we simply see for what values of (x,y) does $x^2 + y^2 = 2y$. Thus, $x^2 + y^2 - 2y = 0$, or $x^2 + (y-1)^2 = 1$. In other words, the projection is simply the circle of radius 1 centered at $(0,1)$:



Now it should be easy to see that

$$\iiint\limits_{S} f(x,y,z)dV = \int_{-1}^{1}\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}}\int_{x^2+y^2}^{2y} f(x,y,z)dzdydx$$

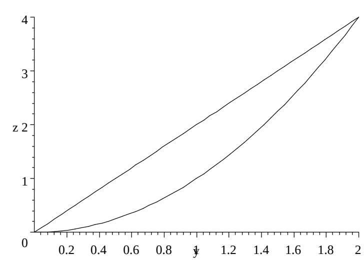
b) $\iiint\limits_{S} f(x,y,z)dV = \int_{[?]}^{[?]}\int_{[?]}^{[?]}\int_{[?]}^{[?]} f(x,y,z)dxdzdy.$

In this integral, we have projected onto the y-z (or $x = 0$) plane. The plane projects simply onto the line $z = 2y$, while the surface projects onto $z = y^2$. To draw a picture, we need to see where these intersect:

$$y^2 = 2y, \text{ or}$$

$$y^2 - 2y = y(y-2) = 0.$$

The points of intersection are thus $(0,0)$ and $(2,4)$. Here's the picture:



And so we have

$$\iiint\limits_{S} f(x,y,z)dV = \int_{0}^{2}\int_{y^2}^{2y}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} f(x,y,z)dxdzdy$$

Finis