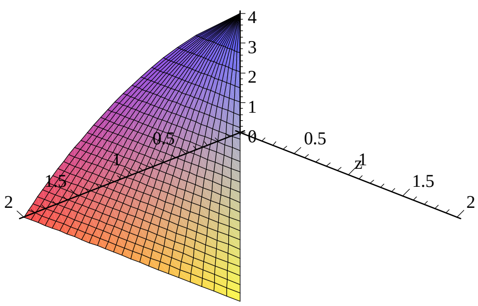
Math 2411 **Quiz Six** **November 28, 2000**

You may use any books, notes, tables, calculators, or computers you wish. Write your answers so that someone other than yourself can understand your exposition. Please do not return this sheet.

Let S be the graph of $z = 4 - x^2$ for $0 \le y \le x$, and $0 \le x \le 2$.

1. Find the area of S.

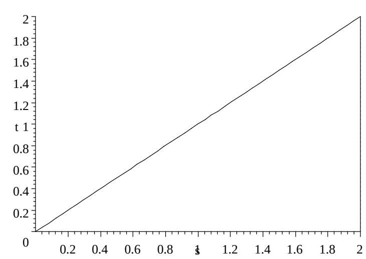
First, a picture:



Now, a vector description of S:

$$\mathbf{r}(s,t) = s\mathbf{i} + t\mathbf{j} + (4 - s^2)\mathbf{k}$$

for $(s,t) \in D$:



Next,

$$\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -2s \\ 0 & 1 & 0 \end{vmatrix} = 2s\mathbf{i} + \mathbf{k}$$

Thus $\left| \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right| = \sqrt{4s^2 + 1}$, and we are ready to compute the area of S.
The area

$$A = \iint_S d\sigma = \iint_D \left| \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right| dA$$

$$= \int_0^2 \int_0^s \sqrt{4s^2+1}\, dt ds = \int_0^2 s\sqrt{4s^2+1}\, ds$$

$$= \frac{(4s^2+1)^{3/2}}{12} \Bigg|_0^2 = \frac{17\sqrt{17}-1}{12}$$

2. Find the flux of $\mathbf{F}(x,y,z) = (x^2y - 3x)\mathbf{i} + \tan(xyz^2)\mathbf{j} + \mathbf{k}$ upwards through S.

Flux is

$$\iint_{\mathbf{S}} \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(s,t)) \cdot \left(\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \right) dA$$

$$= \iint_D [(s^2t - 3s)\mathbf{i} + \tan(st(4-s^2)^2)\mathbf{j} + \mathbf{k}] \cdot [2s\mathbf{i} + \mathbf{k}] dA$$

$$= \iint_D (2s^3t - 6s^2 + 1) dA = \int_0^2 \int_0^s (2s^3t - 6s^2 + 1) dt ds$$

$$= \int_0^2 (s^5 - 6s^3 + s) ds = -\frac{34}{3}$$

Finis