Postage Rates Then and Now and Forever

Here is a table of U. S. First Class postage rates(one ounce) since 1885:

1885	2 cents
1918	3
1919	2
1933	3
1959	4
1963	5
1968	6
1971	8
1974	10
1976	13
1978	15
1981	18
1982	20
1985	22
1988	25
1991	29
1995	32
1999	33
2001	34

Let's fit an exponential function $p(t) = ae^{bt}$ to this data by finding the best least squares linear approximation to the log of the data. This gives a linear approximation for

$$\log(p(t)) = \log a + bt,$$

from which we recover $p(t)$. Here is the log of the data (We have chosen $t = 0$ to be the year 1900.):

$$\begin{pmatrix} -15 & \log 2 \\ 18 & \log 3 \\ 19 & \log 2 \\ 33 & \log 3 \\ 59 & \log 4 \\ 63 & \log 5 \\ 68 & \log 6 \\ 71 & \log 8 \\ 74 & \log 10 \\ 76 & \log 13 \\ 78 & \log 15 \\ 81 & \log 18 \\ 82 & \log 20 \\ 85 & \log 22 \\ 88 & \log 25 \\ 91 & \log 29 \\ 95 & \log 32 \\ 99 & \log 33 \\ 101 & \log 34 \end{pmatrix} \text{ or } A = \begin{pmatrix} -15 & 0.69315 \\ 18 & 1.0986 \\ 19 & 0.69315 \\ 33 & 1.0986 \\ 59 & 1.3863 \\ 63 & 1.6094 \\ 68 & 1.7918 \\ 71 & 2.0794 \\ 74 & 2.3026 \\ 76 & 2.5649 \\ 78 & 2.7081 \\ 81 & 2.8904 \\ 82 & 2.9957 \\ 85 & 3.091 \\ 88 & 3.2189 \\ 91 & 3.3673 \\ 95 & 3.4657 \\ 99 & 3.4965 \\ 101 & 3.5264 \end{pmatrix}$$

$\sum_{i=1}^{19} A_{i,1}A_{i,1} = 102012$ $\quad \sum_{i=1}^{19} A_{i,1} = 1266$ $\quad \sum_{i=1}^{19} A_{i,1}A_{i,2} = 3451.8$

$\sum_{i=1}^{19} A_{i,2} = 44.078$

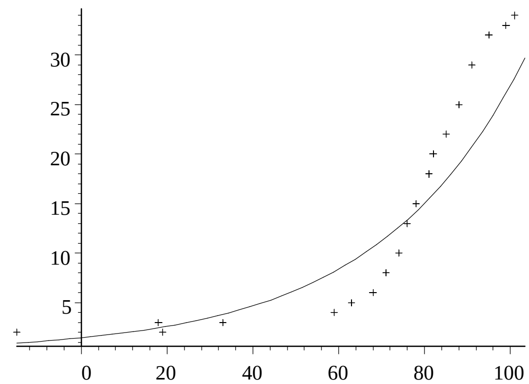
The least squares equations are thus

$$102012M + 1266B = 3451.8$$
$$1266M + 19B = 44.078$$

Solution is: $\{B = 0.37710, M = 2.9157 \times 10^{-2}\}$. Now, for the function $p(t) = ae^{bt}$ we have $B = \log a$ and $M = b$. Thus,

$$p(t) = e^B e^{Mt} = 1.4581e^{0.029157t}$$

Here's a picture of $p(t)$ together with the data points:



Now, we want to know what will happen in 2050. In this year, our model predicts that the price of a one ounce first class letter will be $p(150) = 115.66$ cents, or \$1.16. To find out when the rate will be \$10 per ounce, we solve for t : $p(t) = 1000$. The solution is: $\{t = 223.98\}$. Thus, it will be $1900 + 224$, or 2124 before it costs \$10 to post a first class letter. I can hardly wait.