Math 2401 Quiz 2 Solutions

1. The correct choice is **B**. The surface has two ”peaks” and two ”holes.”

2. Find all points at which the line described by $\mathbf{r}(t) = (1-2t)\mathbf{i} + (2t-1)\mathbf{j} + t\mathbf{k}$ intersects the surface defined by $z = x^2 + 3y^2$.

For a point (x,y,z) to be on the surface, $x, y,$ and z must satisfy the given equation. Thus for a point to be both on the line and on the surface, we must have

$$t = (1-2t)^2 + 3(2t-1)^2, \text{ or}$$

$$t = 4(2t-1)^2 = 16t^2 - 16t + 4, \text{ or}$$

$$16t^2 - 17t + 4 = 0.$$

The famous quadratic formula tells us that the solutions are

$$t = \frac{17 + \sqrt{17^2 - (16)(16)}}{32} = \frac{17+\sqrt{33}}{32}, \text{ and}$$

$$t = \frac{17-\sqrt{33}}{32}.$$

There are thus two points of intersection, and they are

$$\mathbf{r}\left(\frac{17+\sqrt{33}}{32}\right) = \left(1 - 2\frac{17+\sqrt{33}}{32}\right)\mathbf{i} + \left(2\frac{17+\sqrt{33}}{32} - 1\right)\mathbf{j} + \frac{17+\sqrt{33}}{32}\mathbf{k}$$
$$= \left(-\frac{1+\sqrt{33}}{16}\right)\mathbf{i} + \left(\frac{1+\sqrt{33}}{16}\right)\mathbf{j} + \frac{17+\sqrt{33}}{32}\mathbf{k}$$

and

$$\mathbf{r}\left(\frac{17-\sqrt{33}}{32}\right) = \left(-\frac{1-\sqrt{33}}{16}\right)\mathbf{i} + \left(\frac{1-\sqrt{33}}{16}\right)\mathbf{j} + \frac{17-\sqrt{33}}{32}\mathbf{k}$$

.

3. Suppose g is continuous. Find the partial derivatives f_x and f_y, where

$$f(x,y) = \int_x^{xy} g(t)dt.$$

Let $G(t)$ be such that $G'(t) = g(t)$. Then

$$f(x,y) = \int_x^{xy} g(t)dt = G(xy) - G(x).$$

Thus,

$$f_x(x,y) = yG'(xy) - G'(x) = yg(x,y) - g(x), \text{ and}$$
$$f_y(x,y) = xG'(xy) = xg(xy).$$

4. Let

$$f(x,y) = \begin{cases} \frac{y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

a)Find the partial derivative $f_x(0,0)$.

$$f_x(0,0) = \lim_{h\to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{0-0}{h} = 0.$$

$$f_y(0,0) = \lim_{h\to 0} \frac{f(0,0+h) - f(0,0)}{h} = \lim_{h\to 0} \frac{\frac{h^2}{0+h^2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}, \text{ which doesn't exist!}$$

Thus, $f_x(0,0) = 0$, and there is no $f_y(0,0)$.