Math 2401D5 **Quiz One** **January 23, 2001**

You may use any books, notes, tables, calculators, or computers you wish. Be sure to write so that someone other than yourself can understand your exposition.

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1. Find a vector description of the straight line tangent to the curve described by

$$\mathbf{r}(t) = (t^3 + t - 2)\mathbf{i} - t^2\mathbf{j} + (t+1)\mathbf{k}$$

at the point $(-4, -1, 0)$.

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To find a vector description of a line we need a point $\mathbf{p}$ on the line and a vector $\mathbf{v}$ in the direction of the line. We are given $\mathbf{p} = (-4, -1, 0) = -4\mathbf{i} - \mathbf{j}$, so we need only find $\mathbf{v}$. This is easy–it is simply the derivative of $\mathbf{r}(t)$ at $\mathbf{p}$.

First, let's find t_0 so that $\mathbf{r}(t_0) = \mathbf{p}$:

$$\mathbf{r}(t_0) = (t_0^3 + t_0 - 2)\mathbf{i} - t_0^2\mathbf{j} + (t_0 + 1)\mathbf{k} = -4\mathbf{i} - \mathbf{j}$$

I hope it's clear that $t_0 = -1$ does the job. Thus we need $\mathbf{v} = \mathbf{r}'(-1)$.

Now,

$$\mathbf{r}'(t) = (3t^2 + 1)\mathbf{i} - 2t\mathbf{j} + \mathbf{k}.$$

Thus, $\mathbf{v} = \mathbf{r}'(-1) = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. A vector description of our line is $l(t) = \mathbf{p} + t\mathbf{v}$:

$$l(t) = -4\mathbf{i} - \mathbf{j} + t(4\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = (4t - 4)\mathbf{i} + (2t - 1)\mathbf{j} + t\mathbf{k}$$

2. The curve in the picture is described by $\mathbf{r}(t)$. At the point $\mathbf{r}(u)$, the unit tangent $\mathbf{T}$ vector is one of the vectors $\mathbf{A}$ or $\mathbf{B}$ and the principal normal vector is one of the vectors $\mathbf{C}$ or $\mathbf{D}$. At the point $\mathbf{r}(w)$, the vector $\mathbf{E}$ is the unit tangent $\mathbf{T}$ and the principal normal is one of the vectors $\mathbf{F}$ or $\mathbf{G}$. We know that $\mathbf{r}'(t) \neq 0$ for all t. Answer the following questions and explain your answers. If the answer can not be determined from the information given, explain why not.

a)Which of the vectors $\mathbf{A}$ or $\mathbf{B}$ is the unit tangent $\mathbf{T}$ at $\mathbf{r}(u)$?

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The tangent at $\mathbf{r}(w)$ is $\mathbf{E}$ and so we are moving from right to left along the curve. Thus $\mathbf{B}$ is the unit tangent at $\mathbf{r}(u)$.

b)Which is larger, the curvature at $\mathbf{r}(u)$ or the curvature at $\mathbf{r}(w)$?

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The curvature at $\mathbf{r}(u)$ is larger than the curvature at $\mathbf{r}(w)$. The tangent is changing more rapidly at $\mathbf{r}(u)$.

c)Which is larger, u or w?

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The tangent points in the direction of increasing argument t. Thus $u > w$.

d)Which of the vectors **C** or **D** is the principal normal at $\mathbf{r}(u)$?

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The tangent is changing in the direction of **D**. Thus **D** is the principal normal.

e)Which of the vectors **F** or **G** is the principal normal at $\mathbf{r}(w)$?

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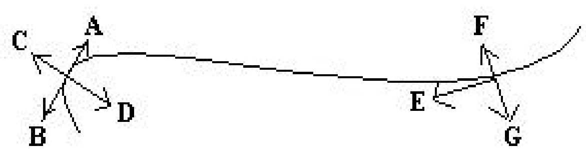
F is the principal normal.



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