Math 2401D5 Quiz Two Solutions

You may use any books, notes, tables, calculators, or computers you wish. Be sure to write so that someone other than yourself can understand your exposition.

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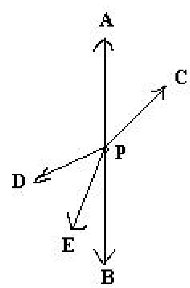
1. In the picture, either **A** or **B** is the gradient of the function f at the point **p**. The directional derivative of f in the direction of the vector **C** is negative.



a) Which of the vectors **A** or **B** is the gradient of the function f at the point **p**? Explain.

The directional derivative in the direction **C** is the scalar product of the gradient and a unit vector in the direction **C**. If this scalar product is to be negative, then the cosine of the angle between the two vectors must be negative–thus the angle between the two must be bigger that $\pi/2$. Thus **B** is the gradient of f.

b) Which is larger, the directional derivative of f in the direction of **D** or the directional derivative of f in the direction of **E**? Explain.

For $0 \leq \alpha \leq \pi$, the $\cos\alpha$ is a decreasing function of α. The angle between **D** and the gradient **B** is larger that the angle between **E** and **B**. Thus the directional derivative in the direction of **E** is larger than that in the direction of **D**.

2. Find all points on the surface $x^2 + 3y^2 + z^2 = 14$ at which the tangent plane is parallel to the plane $x + 6y + z = 50$, or show there are no such points.

The given surface is a level surface of $f(x,y,z) = x^2 + 3y^2 + z^2$. Thus the gradient $\nabla f(x,y,z)$ is normal to the surface, and hence the tangent plane, at (x,y,z). For the two planes to be parallel, their normals must "line up"–that is, one must be a nonzero scalar multiple of the other. In other words, we need to have $\nabla f(x,y,z) = \alpha(\mathbf{i} + 6\mathbf{j} + \mathbf{k})$. This gives us

$$2x\mathbf{i} + 6y\mathbf{j} + 2z\mathbf{k} = \alpha\mathbf{i} + 6\alpha\mathbf{j} + \alpha\mathbf{k},$$

or,

$$2x = \alpha$$
$$6y = 6\alpha$$
$$2z = \alpha$$

and of course

$$x^2 + 3y^2 + z^2 = 14.$$

We simply solve these equations for (x, y, z). First, we get

$$x = \frac{\alpha}{2}, \quad y = \alpha, \text{ and } z = \frac{\alpha}{2}.$$

Substituting these into the last of the equations gives us

$$x^2 + 3y^2 + z^2 = \frac{\alpha^2}{4} + 3\alpha^2 + \frac{\alpha^2}{4} = 14, \text{ or}$$

$$\frac{7}{2}\alpha^2 = 14, \text{ giving us}$$

$$\alpha^2 = 4.$$

And so, $\alpha = 2$ or $\alpha = -2$. For $\alpha = 2$, we have $x = \frac{\alpha}{2} = 1$, $y = \alpha = 2$, and $z = \frac{\alpha}{2} = 1$. Thus, one such point on the surface is $(1, 2, 1)$. For $\alpha = -2$, we get the point $(-1, -2, -1)$.

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