Math 2401 **Quiz Four** **March 15, 2001**

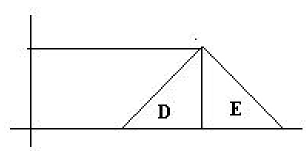
You may use any books, notes, tables, calculators, or computers you wish. Write your answers so that someone other than yourself can understand your exposition. Please do not return this sheet.

1. Let A be the region inside the rectangle with vertices $(0,0),(2,0),(2,1),$ and $(0,1)$. Let B be the region inside the trapezoid with vertices $(0,0),(3,0),(2,1),$ and $(0,1)$. Let C be the region inside the triangle with vertices $(1,0),(3,0),$ and $(2,1)$. Finally, let D be the region inside the triangle with vertices $(1,0),(2,0),$ and $(2,1)$. We know that

$$\iint_A f(x,y)dA = 3,$$
$$\iint_B f(x,y)dA = 2, \text{and}$$
$$\iint_C f(x,y)dA = -5.$$

Find the integral $\iint_D f(x,y)dA,$ or explain carefully why there is insufficient information given.

Let's draw a picture of all this.



Here E is the triangle with vertices $(2,0),(3,0),$ and $(2,1)$.
First, note that

$$\iint_B f(x,y)dA = \iint_A f(x,y)dA + \iint_E f(x,y)dA$$

Thus,

$$\iint_E f(x,y)dA = \iint_B f(x,y)dA - \iint_A f(x,y)dA = 2 - 3 = -1.$$

Next, note that

$$\iint_C f(x,y)dA = \iint_D f(x,y)dA + \iint_E f(x,y)dA, \text{ or}$$

$$\iint_D f(x,y)dA = \iint_C f(x,y)dA - \iint_E f(x,y)dA$$
$$= -5 - (-1) = -4.$$

2. Find the integral

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dydx.$$

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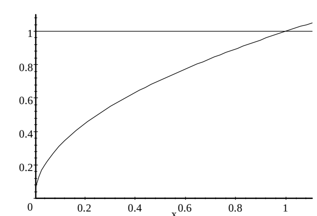
I cannot think of an antiderivative for e^{y^3}, so let's find the two-dimensional integral that lead to this iterated integral and see if changing the order of integration helps.

A picture:



Now,

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3} dydx = \int_0^1 \int_0^{y^2} e^{y^3} dxdy$$
$$= \int_0^1 y^2 e^{y^3} dy = \frac{1}{3} e^{y^3} \Big|_0^1 = \frac{1}{3}(e-1)$$

Finis