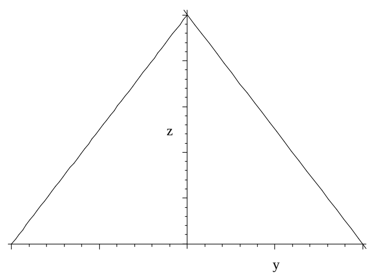
Math 2401 **Quiz Five** **April 3, 2001**

You may use any books, notes, tables, calculators, or computers you wish. Write your answers so that someone other than yourself can understand your exposition. Please do not return this sheet.

1. Let T be the solid bounded by the planes $z = y+1, z+y = 1, x = 0, x = 1,$ and $z = 0$. Evaluate the integral

$$\iiint_T x^2y^2z^2\,dV.$$

Project the solid onto the $y-z$ plane:



We thus integrate first with respect to x, and then integrate over the region R pictured.

$$\iiint_T x^2y^2z^2\,dV = \iint_R \left(\int_0^1 x^2y^2z^2\,dx \right) dA = \int_{-1}^{1}\int_{z-1}^{1-z}\int_0^1 x^2y^2z^2\,dx\,dy\,dz$$

$$= \frac{1}{3}\int_{-1}^{1}\int_{z-1}^{1-z} y^2z^2\,dy\,dz = \frac{1}{9}\int_{-1}^{1} z^2[(1-z)^3-(z-1)^3]\,dz$$

$$= \frac{2}{9}\int_{-1}^{1} z^2(1-z)^3\,dz = \frac{2}{9}\int_{-1}^{1}(z^2-3z^3+3z^4-z^5)\,dz$$

$$= \frac{2}{9}\left(\frac{2}{3}+\frac{6}{5}\right) = \frac{56}{135}$$

2. Find the mass of a wire having the shape of the curve

$$\mathbf{r}(t) = (t^2-1)\mathbf{j} + 2t\mathbf{k},\ 0 \le t \le 1$$

if the density is given by $\rho(t) = \frac{3}{2}t$.

We want

$$\text{Mass} = M = \int_W \rho(t)ds$$

$$= \int_0^1 \frac{3}{2}t|\mathbf{r}'(t)|dt = \int_0^1 \frac{3}{2}t|2t\mathbf{j} + 2\mathbf{k}|dt$$

$$= \int_0^1 3t\sqrt{t^2+1}\,dt = (t^2+1)^{3/2}|_0^1 = 2\sqrt{2} - 1$$

Finis