Math 2401 **Quiz Six** **April 19, 2001**

1. Find a vector description of the surface that results from rotating the curve $y = f(x)$, $0 < a \leq x \leq b$, around the y –axis.

Let s and t be the usual polar coordinates in the x-z plane: $s = r$ and $t = \theta$. Then a vector description $\mathbf{R}(s,t)$ of the surface is

$$\mathbf{R}(s,t) = s\cos t\mathbf{i} + f(s)\mathbf{j} + s\sin t\mathbf{k};$$
$$a \leq s \leq b, \text{ and } 0 \leq t \leq 2\pi.$$

2. Find the flux

$$\iint_{\mathbf{S}} (y\mathbf{i} + \sqrt{xy}\,\mathbf{j}) \cdot d\mathbf{S}$$

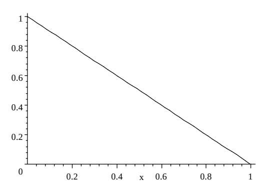
where $\mathbf{S}$ is the surface

$$z = \frac{2}{3}(x^{3/2} + y^{3/2}),\ 0 \leq x \leq 1, 0 \leq y \leq 1 - x$$

with the orientation $\mathbf{n}$ such that $\mathbf{n} \cdot \mathbf{k} > 0$.

First, we get a vector description of $\mathbf{S}$ by simply letting $s = x$ and $t = y$:

$$\mathbf{r}(s,t) = s\mathbf{i} + t\mathbf{j} + \frac{2}{3}(s^{3/2} + t^{3/2})\mathbf{k}, \text{ with domain}$$



We'll need $\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t}$:

$$\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & \sqrt{s} \\ 0 & 1 & \sqrt{t} \end{vmatrix} = -\sqrt{s}\,\mathbf{i} - \sqrt{t}\,\mathbf{j} + \mathbf{k}$$

Observe this normal points in the direction of the specified orientation. Thus

$$\iint_{\mathbf{S}} (y\mathbf{i} + \sqrt{xy}\,\mathbf{j}) \cdot d\mathbf{S} = \iint_D (t\mathbf{i} + \sqrt{st}\,\mathbf{j}) \cdot \left(\frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t}\right) dA$$
$$= \iint_D (t\mathbf{i} + \sqrt{st}\,\mathbf{j}) \cdot (-\sqrt{s}\,\mathbf{i} - \sqrt{t}\,\mathbf{j} + \mathbf{k}) dA$$

$$= \iint_D (-t\sqrt{s} - t\sqrt{s}) = -2 \iint_D t\sqrt{s}\, dA$$

$$= -2 \int_0^1 \int_0^{1-s} t\sqrt{s}\, dt ds = -\int_0^1 \sqrt{s}\,(-1+s)^2 ds = -\frac{16}{105}$$

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