I. Homework to be handed in before 10:06 a.m., Wednesday, January 19

Let S be the set of all vectors (x, y, z) in $\mathbf{R}^3$ such that $x + y + z = 0$.

a)Show that S is a subspace of $\mathbf{R}^3$.
b)Find a basis for S.

II. Homework to be handed in before 10:06 a.m., Wednesday, January 26

Explain carefully what is wrong with **Theorem 3.10** (page 107 of the Apostol textbook).

III. Homework to be handed in before 10:06 a.m., Wednesday, February 3

In the real linear space $C(0,\pi)$ with inner product $(x, y) = \int_0^\pi x(t)y(t)dt$, let

$x_n(t) = \sqrt{\frac{2}{\pi}} \sin nt$.

a)Prove that $S_N = \{x_1, x_2, \ldots, x_N\}$ is an orthonormal set.

b)Find the element y_N of the span of S_N nearest to $f(t) = 1$.

c)Find $\|y_N - f\|^2$.

IV. Homework to be handed in before 10:06 a.m., Wednesday, February 9

Let $\mathbf{T} : \mathbf{R}^4 \to \mathbf{R}^4$ be defined by

$\mathbf{T}(x_1, x_2, x_3, x_4) = (x_1 + x_2 + x_3 + x_4,\ 3x_1 + 2x_2 + 2x_4,\ 7x_1 + 4x_2 - 2x_3 + 4x_4,\ 2x_1 + x_2 - x_3 + x_4)$

a)Find a basis for the null space of $\mathbf{T}$.

b)Find a basis for the range of $\mathbf{T}$.

V. Homework to be handed in before 10:06 a.m., Wednesday, February 16

For each $\mathbf{x} \quad \mathbf{R}^2$, let $\mathbf{T}_\alpha(\mathbf{x})$ be the vector resulting from rotating **x** through an angle α in a counterclockwise direction about the origin.

a)Find the matrix representation with respect to the usual basis {**i**, **j**} of the linear function $\mathbf{T}_\alpha : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ so defined.

b)Find the vector that results from rotating **x** = (14, -23) 60 degrees about the origin (counterclockwise).

c)Find the matrix representation of the composition $\mathbf{T}_\beta \mathbf{T}_\alpha$.

d)Observe that $\mathbf{T}_\beta \mathbf{T}_\alpha = \mathbf{T}_{\alpha+\beta}$, and deduce a familiar trigonometric identity.

VI. Homework to be handed in before 10:06 a.m., Wednesday, February 23

Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ -1 & 2 & 3 & 4 \\ 2 & 2 & 5 & 6 \\ 1 & 3 & 4 & 5 \end{bmatrix}.$$

a)Find the determinant d(A) and prove you have the correct answer. (Telling me what **Matlab** or ***Maple*** or ***Mathematica***, *etc.*, says is **not** a proof.

b)If A has an inverse, find its determinant. Otherwise, explain carefully how you know A is not invertible. (Here also do not simply take the word of some computer program.)

VII. <u>Homework to be handed in before 10:06 a.m., Wednesday, March 29</u>

Let

$$\mathbf{A} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}.$$

a)Find a diagonal matrix similar to **A**.

b)Use the result of part a) to find a square root of **A**. In other words, , find a matrix **R** so that $\mathbf{R}^2 = \mathbf{A}$.

VIII. <u>Homework to be handed in before 10:06 a.m., Wednesday, April 5</u>

1. Exercise #7, page 159

2. Exercise #13, page 226

IX. <u>Homework to be handed in before 10:06 a.m., Wednesday, April 19</u>

Let S be the surface with equation $x^2 + 2yz = 0$, where (x, y, z) are coordinates with respect to the standard basis for $\mathbf{R}^3$. Find an orthonormal basis $E = \{u_1, u_2, u_3\}$ for $\mathbf{R}^3$ so that the coordinates $(\tilde{x}, \tilde{y}, \tilde{z})$ of the points of S with respect to E satisfy an equation in which there are no “cross product” terms—that is, the equation involves only $\tilde{x}^2$, $\tilde{y}^2$, and $\tilde{z}^2$. Identify and sketch the surface.

X. **<u>Homework to be handed in before 10:06 a.m., Wednesday, April 26</u>**

1. Suppose the square matrix $\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ has the property that

$$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \text{ for each } i = 1, 2, \ldots, n.$$

Explain how you know $\boldsymbol{A}$ is invertible.

2. Suppose $\mathbf{A}$ and $\mathbf{B}$ are n x n matrices, and suppose $\mathbf{A}$ is invertible and $|\lambda| < 1$ for every eigenvalue λ of $\mathbf{A}^{-1}\mathbf{B}$. Explain how you know the matrix $\mathbf{A} + \mathbf{B}$ is invertible.