To be handed in no later than 9:20 a.m., Monday, July 19.

1. Suppose the square matrix $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$ has the property that $|a_{ii}| > \prod_{\substack{j=1 \\ j=i}}^{n} |a_{ij}|$ for each i = 1, 2, ..., n.

Explain how you know A is invertible.

2. a) Let

$$A = \begin{array}{cc} 1 & 2 \\ 3 & 4 \end{array}.$$

Find the condition number cond(A) that is generated by the norm | |.

b) Give an example of a matrix A such that cond(A) < 1, or explain carefully why there is no such matrix.

- **3.** Suppose **A** and **B** are n x n matrices, and suppose **A** is invertible and $||\mathbf{A}^{-1}\mathbf{B}|| < 1$. Explain how you know the matrix $\mathbf{A} + \mathbf{B}$ is invertible.
- 4. Find as small an upper bound as you can for the magnitude of the error incurred using linear interpolation in a table of $\tan x$ for $0 \times 45^{\circ}$ in which there are equally spaced entries one *degree* apart.