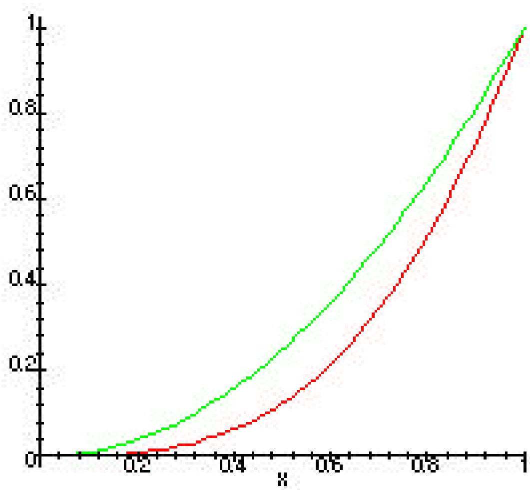
+++++++++++

1. For the region R, we have

$$\iint_R (x+y^2)\,dA = \int_0^1 \int_{\sqrt{y}}^{y^{1/3}} (x+y^2)\,dx\,dy.$$

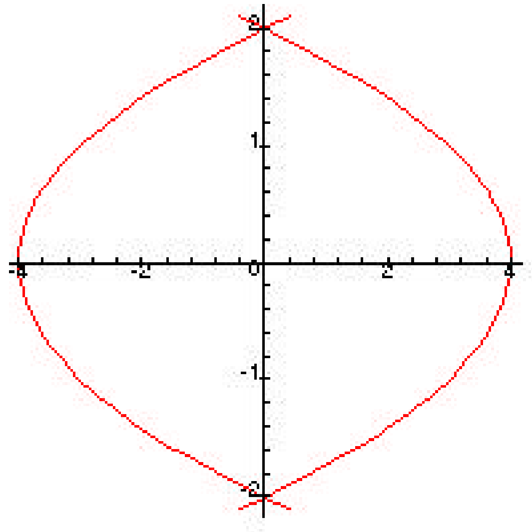
a) Sketch the region R.



b) Give an iterated integral for $\iint_R (x+y^2)\,dA$ in which the first, or "inside" integration is with respect to y. [You need not evaluate the integral.]

$$\iint_R (x+y^2)\,dA = \int_0^1 \int_{x^3}^{x^2} (x+y^2)\,dy\,dx$$

2. The plate P has the shape of the region bounded by the curves $x = 4 - y^2$ and $x = y^2 - 4$. The density of the plate is given by $\rho(x,y) = y + 5$. Find the centerof mass of P.



$$\tilde{x} = \frac{\iint_P x\rho(x,y)\,dA}{\iint_P \rho(x,y)\,dA} \quad \text{and} \quad \tilde{y} = \frac{\iint_P y\rho(x,y)\,dA}{\iint_P \rho(x,y)\,dA}.$$ Let's evaluate the integrals:

$$\iint_P \rho(x,y)\,dA = \int_{-2}^{2}\int_{y^2-4}^{4-y^2}(y+5)\,dx\,dy = \int_{-2}^{2}(y+5)(4-y^2-y^2+4)\,dy = 2\int_{-2}^{2}(y+5)(4-y^2)\,dy = \frac{320}{3}$$

$$\iint_P x\rho(x,y)\,dA = \int_{-2}^{2}\int_{y^2-4}^{4-y^2}x(y+5)\,dx\,dy = \frac{1}{2}\int_{-2}^{2}[(4-y^2)^2-(y^2-4)^2](y+5)\,dy = \frac{1}{2}\int_{-2}^{2}0\,dy = 0.$$

$$\iint_P y\rho(x,y)\,dA = \int_{-2}^{2}\int_{y^2-4}^{4-y^2}y(y+5)\,dx\,dy = 2\int_{-2}^{2}y(y+5)(4-y^2)\,dy = \frac{256}{15}.$$

Hence,

$$\tilde{x} = 0 \ \text{ and } \ \tilde{y} = \frac{256/15}{320/3} = \frac{4}{25}$$