**Math 4320A** **Final Examination** **Summer 1997**

You may use any books, notes, or calculators you wish. *Fortuna vobiscum.*.

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**1.** Find all *z* such that $e^z = 1 + i$ .

**2.** For the given function $u$ , find a function $v$ such that $f(z) = u(x,y) + iv(x,y)$ is entire, or explain carefully why there is no such $v$.

a) $u(x,y) = x^2 - x - y^2$ b) $u(x,y) = x^2 - x - y^3$

**3.** Let S be the square with vertices (0, 0), (1, 0), (1, 1), and (0, 1). Find the maximum value of $\left|\frac{z-1}{z+5}\right|$ for z in S.

**4.** Let $f(z) = \frac{1}{z^2(z-i)}$ .

a) Find a Laurent series expansion in powers of z for *f*, and specify the region for which it is valid.

b) Find another Laurent series expansion in powers of z for *f*, and specify the region for which it is valid.

c) Let C be the circle C = {z : |z| = 1/2 } oriented positively. Find $\int_C \frac{1}{z^2(z-i)} dz$ .

d) Let C be the circle C = {z : |z| = 3/2 } oriented positively. Find $\int_C \frac{1}{z^2(z-i)} dz$ .

**5.** Find $\int_0^\infty \frac{1}{x^4+1} dx$.

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