Math 4320A - Winter 1999 - Homework to be handed in.

1. Monday, January 11:

Find all z such that $\bar{z}^2 + z + 1 = 0$.

2. Wednesday, January 13:

a)Suppose that $\arg\left(\frac{z}{w}\right)$ is an argument of $\frac{z}{w}$. Prove there are arguments $\arg z$ and $\arg w$ of z and w, respectively, such that $\arg\left(\frac{z}{w}\right) = \arg z - \arg w$.

b)Is it always true that $\operatorname{Arg}\left(\frac{z}{w}\right) = \operatorname{Arg} z - \operatorname{Arg} w$? Prove your answer.

3. Wednesday, January 20:

Is the function f given by

$$f(z) = \begin{cases} \frac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

differentiable at $z = 0$? Explain.

[N.B. This is Exercise 9, Chapter Two.]

4. Friday, January 22:

Do the real and imaginary parts of the function defined in the previous problem satisfy the Cauchy-Riemann equations at $z = 0$? Explain.

5. Monday, January 25:

Find all z such that $\sin z = 2$, or explain carefully why there are none.

6. Wednesday, February 3:

a)Let $F(z) = \log z$, $0 < \arg z < 2\pi$. Show that $F'(z) = \frac{1}{z}$.

b)Let $G(z) = \log z$, $-\frac{3\pi}{4} < \arg z < \frac{5\pi}{4}$. Show that $G'(z) = \frac{1}{z}$.

c)Let C_1 be a curve in the right half-plane $D_1 = \{z : \operatorname{Re} z \geq 0\}$ from $-i$ to i. Find

$$\int_{C_1} \frac{1}{z} dz .$$

d)Let C_2 be a curve in the left half-plane $D_2 = \{z : \operatorname{Re} z \leq 0\}$ from $-i$ to i. Find

$$\int_{C_2} \frac{1}{z} dz .$$

[N.B. This is Exercise 8, Chapter Four.]

7. Monday, February 8:

Let $f(z) = c^z$, $-\pi < \arg c < \pi$; and $g(z) = c^z$, $-\frac{\pi}{2} < \arg c < \frac{3\pi}{2}$.

a)Find all c for which $f(z) = g(z)$ for all z.

b)For what c is it true that $f(a)f(b) = f(a+b)$?

c)Suppose c is such that there is at least one z for which $f(z) \neq g(z)$. For what values (if any) of a and b is it true that $f(a)g(b)=f(a+b)$? For what values of a and b does $f(a)g(b)=g(a+b)$?

8. Friday, February 12:

Evaluate the integral $\int_C f(z)\,dz$, where $f(z) = z^{\frac{1}{2}}$, $\frac{\pi}{2} < \arg z < \frac{5\pi}{2}$, and C is any curve in the lower half-plane from -1 to 1.

9. Monday, February 15:

For each of the individuals described below, give the date and place of birth, and if they are deceased, the date and place of death. If they are still alive, give the city in which they currently reside.

a) The person for whom Cauchy's Theorem is named.

b) The person for whom Morera's Theorem is named.

10. Friday, February 19:

Let C be the circle $|z + i| = 2$ positively oriented. Find the integral

$$\int_C \frac{1}{(z^2 + 4)^2}\,dz .$$

11. Monday, March 1:

Find the limit $\sum_{j=1}^{\infty} jz^j$. What is the radius of convergence of the series?

12. Friday, March 5:

Give two Laurent series in powers of z for the function f defined by

$$f(z) = \frac{1}{z(z^2 + 4)}$$

and specify the regions in which the series converges to f .