Math 4320A **Final Examination** **June 13, 1997**

You may use any books, notes, or calculators you wish. Write your answers so that someone other than yourself can understand your exposition. *Fortuna vobiscum.*.

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1. Find all points at which the function has a derivative and find all points at which it is analytic. Explain.

a) $f(z) = 2x(1-y) + i(x^2 - y^2 + 2y)$. b) $f(z) = e^y e^{-ix}$.

2. Let $f(z) = z^i$, where z^i denotes the principal branch of z^i. Find

$$\int_L f(z)\,dz,$$

where L is the line segment from (1, 0) to (2, 0). [Give your answer in rectangular form: $a + ib$, where a and b are real.]

3. . Let $f(z) = \dfrac{1}{z^3(z-i)}$.

a) Find a Laurent series expansion in powers of *z* for *f*, and specify the region for which it is valid.

b) Find another Laurent series expansion in powers of *z* for *f*, and specify the region for which it is valid.

4. Let C be the circle $C = \{z : |\, z \,| = 1/2\}$ oriented positively. Find $\int_C \dfrac{1}{z^4(z-i)} dz$.

5. Find the Taylor series representation of $f(z) = \dfrac{1}{1-z}$ in powers of $z + i$. For what values of z is this representation valid? Explain.

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