

You may use any books, notes, tables, or calculators you wish. Be sure to write so that someone other than yourself can understand your exposition. Please give exact answers—no decimal or other approximations. *Fortuna vobiscum.*

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1. Sketch the set of points, if any, at which the function defined by  $f(z) = x^5 + iy^3$  has a derivative. Where is this function analytic? Explain.

2. Let  $f(z) = \frac{1}{z^3(z+2i)}$ .

- a) Find a Laurent series expansion in powers of  $z$  for  $f$ , and specify the region for which it is valid.  
b) Find another Laurent series expansion in powers of  $z$  for  $f$ , and specify the region for which it is valid.

4. Let  $C$  be the circle  $C = \{z : |z| = 1/2\}$  oriented positively. Find  $\oint_C \frac{1}{z^3(z+2i)} dz$ .

5. Give an example showing that the statement is not always true, or explain how you know it is always true:

a)  $\text{Log } z^n = n \text{Log } z$ .

b)  $1^z = 1$

c)  $e^{\log z} = z$

d)  $\log e^z = z$

6. Find a circle inside which are found all solutions of the equation

$$z^7 + 2z^6 + 4z^5 + iz^4 - z^3 + 2z^2 + (1+2i)z - 1 = 0,$$

and explain your answer.